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The impact of neighbourhood size on the accuracy of cellular automata-based urban modelling

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Abstract: Cellular automata are discrete dynamic models in which behaviour is specified in terms of local relations. This technique has recently been advantageously applied to modelling of the urban development process. However, the behaviour of the model is affected by spatial scale, including cell size and neighbourhood extent. Therefore, it is important to examine the impacts of various neighbourhood scales on the model's behaviour and outcome. In this paper we configured a cellular automata model of urban growth in Sydney, Australia, using three different neighbourhood scales: a small neighbourhood of 1.5 cells radius, a moderate neighbourhood of 2.5 cells radius and a large neighbourhood of 3.5 cells radius, all with a fixed cell size of 250 metres. The moderate neighbourhood scale of 2.5 cells radius was found to best reflect those local mechanisms that have the most direct impact on urban development in Sydney. Hence this paper provides a useful reference in the search for a neighbourhood size that is suitable for cellular automata-based modelling of the processes of urban development.

Key Words: Urban development; cellular automata; neighbourhood scale; Sydney.

1. Introduction

Cellular automata (CA) models have been applied to the simulation of urban growth by many researchers, such as Couclelis (1985;1989;1997), White et al. (1993; 1994; 1997a; 1997b), Itami (1994), Batty (1997; 1998), Batty et al. (1994; 1997; 1999), Cecchini (1996), Clarke et al. (1997; 1998), Wu (1996; 1998a; 1998b; 1998c), Liu et al. (2003; 2004; 2005) and Lau et al. (2005). These models are characterised by phase transitions which generate complex spatial patterns through simple transition rules. As such, cellular automata are ideally suited to modelling the spatio-temporal process of urban growth (Clarke et al. 1998; Batty 1995).

In a cellular automata system, space is divided into regular spatial units called cells with time progressing in discrete steps. Each cell in the system maintains one of a finite number of states. The state of each cell is updated according to local rules, that is, the state of a cell at a given time is determined by the state of the cell itself and the states of cells in its neighbourhood at a previous time step through a set of local transition rules (Wolfram 1984).

For instance, the earliest application of cellular automata was John Conway's 'Game of Life', which was constructed as a two-state, eight-cell neighbourhood and two-dimensional grid (Gardner 1972). The state of each cell in the grid can be either dead or live. A cell can survive, die or give birth in successive generations according to a number of rules as follows:

- Survival: a live cell with two or three live neighbours survives into the next generation,
- Death: a live cell with less than two or more than three live neighbours dies either of isolation or of overcrowding, and

- Birth: a dead cell with exactly three live neighbours becomes alive in the next generation.

Using these simple rules at the local level, the model generates very complex global patterns as different cells die, survive or give birth in successive generations (Figure 1).

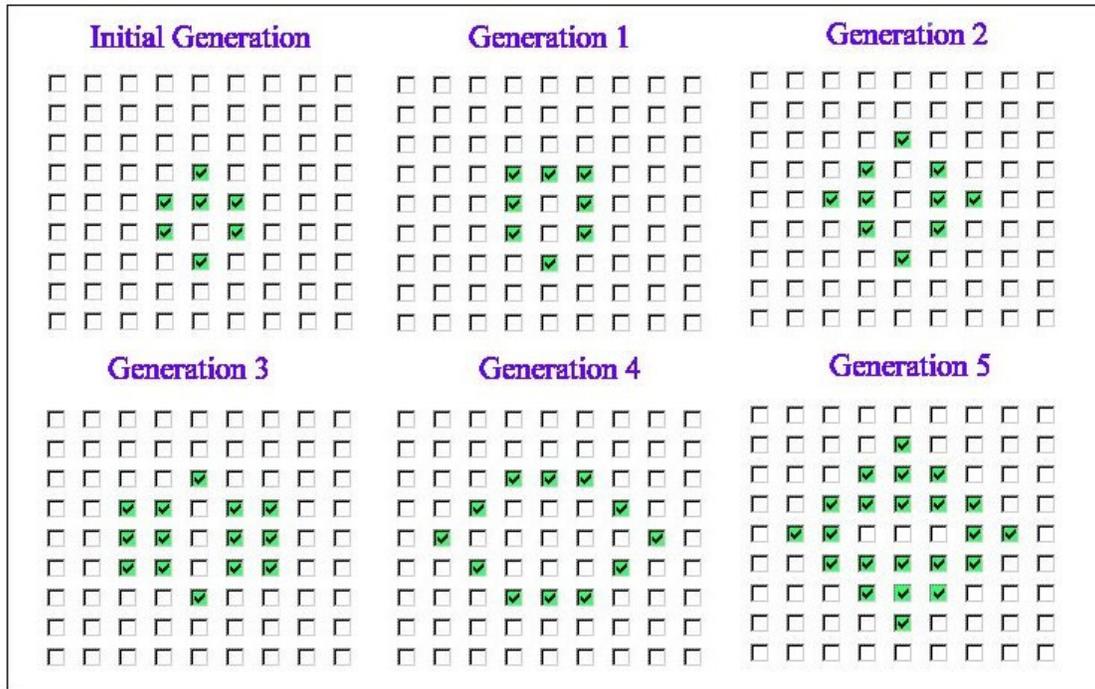


Figure 1 - A sample simulation based on Conway's 'Game of Life'

There are four basic elements of a cellular automaton. They include the cells, the state of cells, the neighbourhood and the transition rules. For an urban system, the cells are typically defined as a regular tessellation of an urban space, with the state of cells representing either urban or non-urban or any specific land use types. The state of cells changes according to some local transition rules which are effective within a certain neighbourhood. The neighbourhood is a region with an impact on the transition of the cell state. Obviously, the spatial scales of the cellular automata, namely, the cell size and the scale of neighbourhood are critical parameters to the configuration of the model.

Various cell sizes have been experimented with in previous research based on cellular automata theory. For instance, Wu (1996; 1998b; 1998c) used both 28.5-metre and 200-metre cells to model an area of 224 square kilometres. By contrast, White et al. (1993) constructed a cellular automata model with a 500-metre grid size to simulate the urban land use patterns in a set of US cities; and they also modified their model to a 'high-resolution scale', with a 250-metre grid size, to simulate urban land use dynamics in the city of Cincinnati, Ohio (White et al. 1997b:323).

In addition, Clarke and colleagues used a basic grid of 300-metre cells for the San Francisco Bay area. However, while applying their model to the Washington/Baltimore region, calibrations were undertaken at resolutions of 210, 420, 840 and 1680 metres respectively (Clarke et al. 1998). This work shows that although not all rules or factors are sensitive to the change of the cell scale, the scale of cells does, however, have an impact on the results of the simulation, especially in relation to certain factors, such as roads and slopes. The authors suggested a hierarchical approach in calibrating the model by 'first using coarse data to investigate the scaling nature of each parameter in a different city setting, then scaling up once the best data ranges are found' (Clarke et al. 1998:710).

According to the principles of cellular automata, the global behaviour of a self-organizing system is governed by locally defined transition rules. For an urban system, a fundamental question is to what extent urban development is a locally specified process (Wu 1996). Some factors, such as slope and elevation of land, affect urban development in a small area base. Others such as urban planning and the transportation networks are global controls over the whole area. Moreover, developments in information technology and telecommunications also have significant consequences for the patterns and processes of urban change throughout the world (Herbert et al. 1997). These factors affect urban development in a universal way.

In practice, both small and large neighbourhood scales have been applied to models of urban development. The former, being a nine-cell Moore Neighbourhood as was applied in Clarke et al. (1998), Wu (1996; 1998a; 1998b; 1998c) and Clarke et al. (1997), and the latter, being 112 cells surrounding the cell in question, as was applied in White et al. (1993; 1994). While there is no particular validation of the scales of neighbourhood in the CA-based urban models, the tendency is leaning towards an extended neighbourhood (Menard et al. 2005). This tendency is clearly obvious in CA based urban research compared to the applications in the natural sciences (Batty et al. 1994). The justification behind this larger neighbourhood scale is probably because of the difficulty in justifying transition rules in behavioural terms (Wu 1996) and the existence of distance-decay effects of the neighbouring cells to the central cell in question (White et al. 1993; White et al. 1994; Wu 1996).

Regardless of the scale of the neighbourhood, the type of neighbourhood also has significant impacts on the behaviour of a cellular automaton. Li et al. (2000) show that the use of a rectangular neighbourhood, such as the Moore Neighbourhood, might produce significant distortions between cells at different directions from a circular object. In this case, a circular neighbourhood is more accurate in representing the cells in all directions of the neighbourhood (Li et al. 2000).

Moreover, although the configuration of neighbourhood scale and its impact on the model's behaviour have not been studied systematically, there have been at least two attempts. Firstly, Menard et al. (2005) demonstrated that while the simulation result of a geographical cellular automata model is sensitive to the spatial scale of cells and neighbourhood, the choice of a neighbourhood scale is less influential on simulation results; it becomes more sensitive only when a large scale neighbourhood with a circular neighbourhood of approximately a 5 cell radius is used, or when a medium-sized neighbourhood with a circular neighbourhood of an approximately 2 cell radius is applied in combination with a large cell size of 1000 metres.

Secondly, Kocabas et al. (2006) analysed the sensitivity of a CA model under different neighbourhood sizes and types. They showed that their CA model was sensitive to:

1. changes in spatial resolution when neighbourhood size and type are kept constant;
2. changes in spatial resolution when neighbourhood size is kept constant but neighbourhood type is being changed;
3. changes of neighbourhood size when spatial resolution and the neighbourhood type are kept constant; and
4. changes of neighbourhood type when spatial resolution is kept constant but the neighbourhood size is being changed

However, both researches failed to address the mechanism underpinning the selection of the neighbourhood scales and the configuration of a neighbourhood scale so that the CA models can generate scenarios that best mimic the actual process of urban development.

This paper presents research on a fuzzy-constrained cellular automata model of urban development, and it focuses on the evaluation of the impact of various neighbourhood scales on the behaviour of the model and its outcome by isolating the neighbourhood scale from

other factors such as the spatial scale of cells (cell size) and the transition rules. Results generated from the CA model under different neighbourhood configurations were analysed and compared against the actual urban development which enables the selection of a suitable neighbourhood scale to best match the actual urban development process. The following section presents the modelling framework and the configuration of three different neighbourhood scales for the study area in Sydney, Australia. This was followed by sensitivity analysis and discussion on results generated by the model, which enabled conclusions to be established on the effect of neighbourhood scales on urban modelling based on cellular automata approach.

2. The Modelling Framework

2.1 Study area and data

The metropolitan area of Sydney, Australia, as bounded by the Nepean-Hawkesbury River and its tributaries, was selected as the study area. The urban area of Sydney has been expanding rapidly in space and over time during the last three decades. A GIS based spatial visualisation on the urban growth of Sydney is available in Liu (1998).

Geographical data was collected and processed in GIS. Census data from the Australian Bureau of Statistics (ABS) was used to define the state of cells of the urban cellular automata model. As urban development is a continuous process in space and over time, it would be difficult and unrealistic to classify the state of cells into either urban or non-urban based on crisp set theory. Instead, a fuzzy set approach was applied to define the state of cells (Liu et al., 2004).

A linear membership function based on population density was used to define the state of cells representing the fuzzy process of non-urban to partly-urban and then urban transitions. The degree of membership ranges from 0 to 1 inclusive, with 1 being a fully developed urban state and 0 a non-urban rural state. A number within the range of 0 to 1 indicates that the cell is currently undergoing transition from a non-urban to urban state. For instance, if a cell has a membership grade of 0.8, it indicates that the cell has been partly urbanised and has developed to a higher extent than a cell with a membership grade of 0.3 in the urban development process.

The linear membership function was applied to process the census data in 1971, 1986, 1991 and 1996. The data was then used to illustrate the actual urban scenarios in the specified time period and also to define the initial state of cells of the CA model of urban development.

In addition to the use of the census data, other data sets such as the topographic data of the area, data illustrating the transportation networks and the various urban planning schemes were also collected and processed in GIS, which were used in defining the transition rules of the CA-based urban model in Sydney (Liu et al., 2004).

2.2 A fuzzy-constrained CA model of urban development

The fuzzy-constrained CA model adopted in this research was originally designed by Liu et al. (2003) and it was subsequently calibrated to simulate the urban development of Sydney, Australia (Liu et al. 2004; 2005). In order to evaluate the impact of neighbourhood scale on the model's behaviour, the impact of other factors, such as the cell scale and transition rules were isolated by keeping the cell scale constant and applying transition rules universally to all cells. In this case, the cell scale was 250 meters and transition rules were applied representing land availability, land released or planned to be released for urban development, land contiguity and land accessibility, with the latter being represented by slope, terrain and transport networks (defined and calibrated in Liu et al., 2005).

According to Herbert et al. (1997), the process of urban development follows a logistic curve over time. However, the driving factors for such development are multi-faceted, representing

constraints of the physical environment, social-economic conditions as well as institutional controls. Hence for Sydney, five factors representing urban natural growth, slope constraints, transportation support, terrain and coastal proximity attractions as well as land availability and urban planning support were progressively identified and calibrated into the cellular automata model using fuzzy-logic constrained transition rules.

The model can be written as:

$$S_{ij}^{t+1} = f(p_{ij}^N, s_{ij}^N, t_{ij}^N, h_{ij}^N, k_{ij}^N) \quad (1)$$

where : S_{ij}^{t+1} is the state of a cell at location i, j at time $t+1$;
 f is a logistic function of urban development, the shape of which is controlled by five transition factors at time t ;
 N is the size of the neighbourhood;
 p_{ij}^N is the contiguity factor, that is, it represents the propensity of a cell for development and the support for such development it can receive from its neighbourhood;
 s_{ij}^N is a factor representing slope constraints;
 t_{ij}^N is a factor representing support of transportation networks on the urban development;
 h_{ij}^N is a factor representing terrain and coastal proximity attractions on the urban development; and
 k_{ij}^N is a factor representing areas planned for urban development in various urban planning schemes to release slope constraints and reinforce transportation support and terrain and coastal proximity attractions.

Transition of cells from one state to another was controlled by the effects of the five factors defined above, constrained by fuzzy logic within the defined neighbourhood. For instance, continued development occurs in partly-urban cells under certain circumstances, such as when there are strong driving forces for urban development from the neighbourhood (p_{ij}^N), or there is proximity to a transportation network (t_{ij}^N), or there is a loss of physical land constraints (s_{ij}^N).

A cell may undergo:

- *rapid development* if there are other factors in favour of such development,
- *slow development* if there are other constraints for the development or
- *new development* on un-developed non-urban cells if there are strong driving forces in the neighbourhood (p_{ij}^N) to lead to the development of these cells.

Note that other factors such as slope, transport or urban planning control may also affect the speed of the development. Also, *no development* applies to cells located in areas excluded from urban development, such as water bodies, national parks or natural reserves, or if there were no strong driving forces within the defined neighbourhood that could lead to new or continued development of the cell.

2.3 Three neighbourhood scales

Owing to the distance-decay effect of the neighbouring cells on the central cell in question, the use of a rectangular neighbourhood in CA-based urban models can produce distortion on an object of any shape. This distortion is especially significant when a large neighbourhood size applies, which can be eliminated by applying a circular neighbourhood (Li and Yeh, 2000).

Three different sizes of circular neighbourhoods were defined by specifying a radius in cells from the centre of the processing cell. This radius ranges from 1.5 to 3.5 cells, or 375 metres to 875 metres on the ground (Figure 2). Any cell centre encompassed by the circle was included as a neighbour of the processing cell. The neighbourhood scale of a 2.5 cells in radius was termed a moderate neighbourhood, while the other two scales of 1.5 and 3.5 cells in radius were termed small and large neighbourhood respectively.

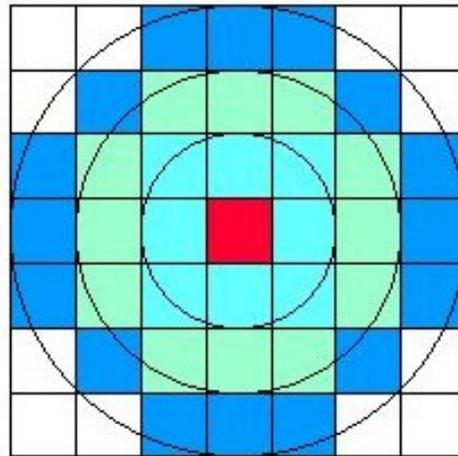


Figure 2 - The three scales of neighbourhoods. The cell in red is the cell in question; a small neighbourhood is shown by turquoise cells; a moderate neighbourhood is shown by turquoise and green cells and a large neighbourhood is shown by turquoise, green and blue cells.

3. Assessment of the Model's Accuracy and Results

The error matrix method is a common means of assessing raster data from two different sources and compares the relationship between the two data sources on a site-by-site basis (Story et al., 1986; Lillesand et al., 1994; Jensen, 1996). However, the limitation of using the standard error matrix approach is that it may not necessarily capture the qualitative similarity of patterns between the two datasets (Power et al., 2001; Liu et al., 2004). For a simulation model of urban development, it is the pattern of the cell states that has functional significance in the urban development context.

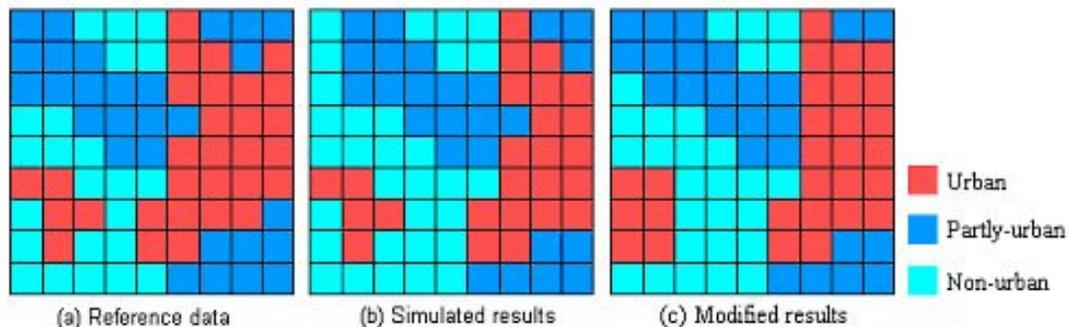


Figure 3 - A comparison amongst the reference data (a), the simulated results (b) and the majority of cell states within a 3 by 3 neighbourhood of the simulated results (c)

Accordingly, to enhance the pattern effect of the simulated results, a majority state (the state that appears most frequently) within a 3 by 3 neighbourhood of each cell in question, was computed and assigned to the corresponding cell (Figure 3). If there were more than one majority state in the neighbourhood, the one with the same state as the output cell was

assigned to the processing cell. By using the majority state of the cell within the neighbourhood rather than the state of the cell itself in preparing the error matrix, the similarity of patterns between the model's results and the reference data can be identified more clearly.

3.1 The Modified Error Matrix and Kappa Analysis

To improve the interpretation of the error matrix, a Kappa coefficient analysis was conducted which yields a K_{hat} to measure the difference between the observed agreement of two maps and the agreement that might be attained by the chance matching of the maps (Campbell, 1996). The K_{hat} coefficient is computed as

$$K_{\text{hat}} = \frac{M \sum_{i=1}^r x_{ii} - \sum_{i=1}^r (x_{i+} \times x_{+i})}{M^2 - \sum_{i=1}^r (x_{i+} \times x_{+i})} \quad (2)$$

where: r is the number of categories in the error matrix;
 x_{ii} is the number of observations in row i and column i ;
 x_{i+} is the total number of observations in row i (shown as marginal total to the right of the matrix);
 x_{+i} is the total number of observations in column i (shown as marginal total to the right of the matrix); and
 M is the total number of observations included in the matrix.

More details on the modified error matrix approach and Kappa coefficient analysis can be found in Liu et al. (2004).

3.2 Results and Discussion

In order to investigate the impact of the neighbourhood scale on the model's performance and outcome, the model was computed using the Sydney datasets with the transition rules and the three different neighbourhood scales outlined above. The starting date of the model was set to 1971 and the ending date to 1996. Each iteration of the model represents one year in the temporal dimension.

A number of different urban scenarios of Sydney were generated by the model, with the last iteration of the model results for year 1996 illustrated in Figure 4 (b, c, d). The simulation accuracy of the model was assessed by comparing the simulated urban scenarios against the actual urban development of Sydney in 1996 (Figure 4 a) using the modified error matrix and Kappa coefficient analysis approach.

Comparing the model's results with the actual urban development of Sydney visually, the model with a small neighbourhood (radius = 1.5) generated less development than the actual urban development of Sydney. With a large neighbourhood configuration (radius = 3.5), the model generated more development than the actual urban development on the ground.

Statistically, the percentages of cells in the urban category under the three neighbourhood scales were close to the values in the actual urban development, and their user's and producer's accuracy in this category were similar (Table 1). However, significant differences existed in the partly-urban and non-urban categories. With the small neighbourhood configuration, the partly-urban cells made up only 7.0 per cent of the total cells in 1996, leaving 65.6 per cent of the cells undeveloped. On the other hand, when the model was

configured with the large neighbourhood scale, 12.6 per cent of the total cells were developed into partly-urban, leaving only 58.5 per cent of the cells undeveloped.

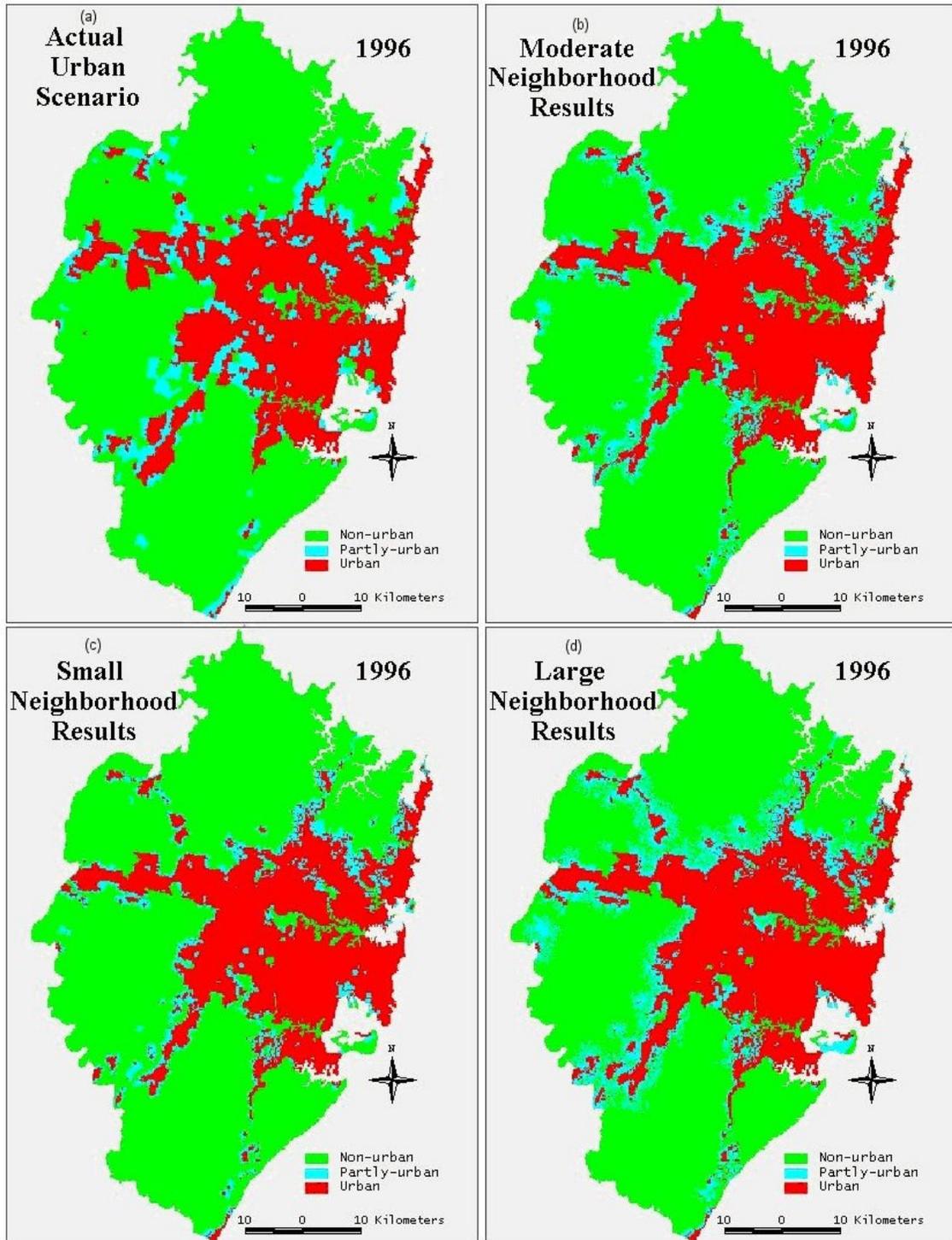


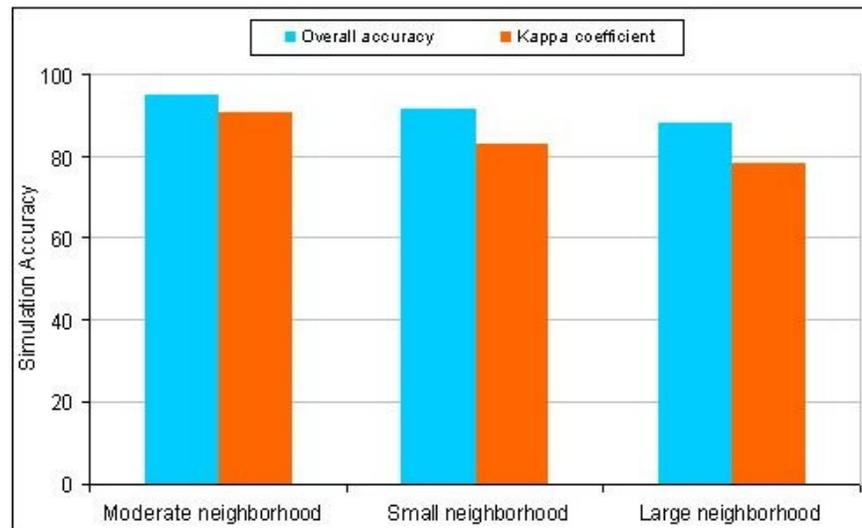
Figure 4 - Results of the simulation of Sydney's growth in 1998 under three different neighbourhood settings, with a comparison to the actual urban growth.

Consequently, the model with the small neighbourhood scale generated an overall accuracy of 91.3 per cent, while the one with the large neighbourhood generated an overall accuracy

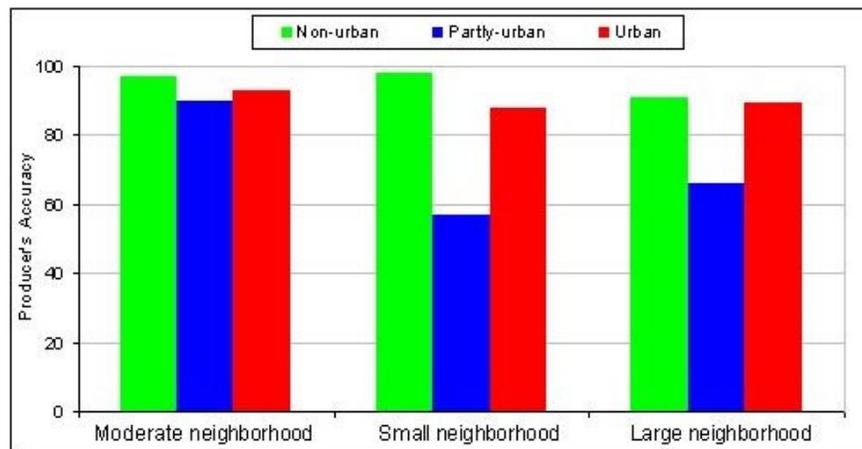
of only 88.1 per cent. Owing to the discrepancy of individual accuracy in the partly urban and non-urban categories from both the producer's and the user's perspective, the K_{hat} coefficient of these two scenarios was only 82.8 per cent and 78.0 per cent respectively. By contrast, when the model was configured with the moderate neighbourhood, it generated results close to the actual urban development scenario, resulting in an overall accuracy of 95.0 per cent and a K_{hat} coefficient of 90.5 per cent (Table 1 and Figure 5).

		Actual urban scenario	Moderate neighbourhood	Small neighbourhood	Large neighbourhood	
Percentage of cells in each category (%)	Non-urban	62.6	61.8	65.6	58.5	
	Partly-urban	9.2	9.9	7.0	12.6	
	Urban	28.2	28.3	27.4	28.9	
	Total	100	100	100	100	
Simulation accuracies (%)	Overall accuracy		-	95.0	91.3	88.1
	Producer's accuracy	Non-urban	-	96.6	98.0	90.8
		Partly-urban	-	89.8	56.5	66.1
		Urban	-	93.1	87.9	89.4
	User's accuracy	Non-urban	-	97.9	93.5	97.2
		Partly-urban	-	82.8	73.8	48.3
		Urban	-	92.9	90.5	87.1
K_{hat} coefficient		-	90.5	82.8	78.0	

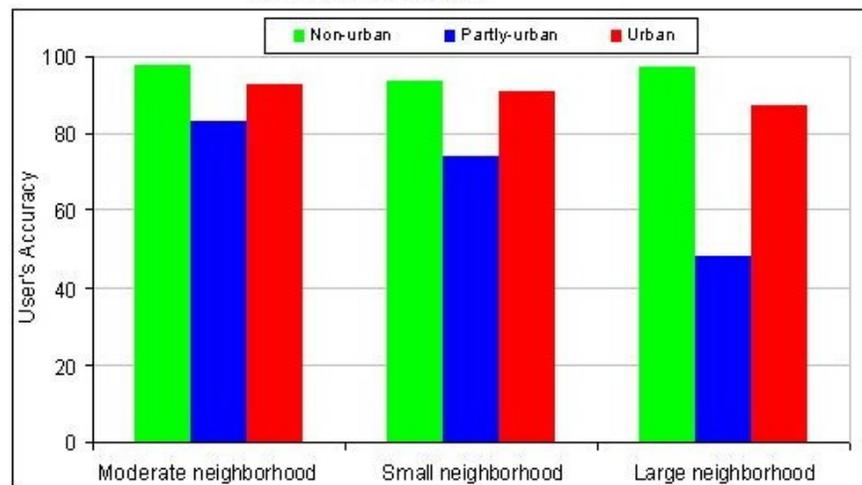
Table 1 - Comparison of the model's outputs in 1996 under various neighbourhood scales. The actual urban scenario refers to the actual 1996 urban development of Sydney. The moderate neighbourhood refers to when the neighbourhood scale of the cellular automaton was configured with a radius of 2.5 cells ($r = 2.5$), whereas the small and large neighbourhoods refer to radiuses of 1.5 cells ($r = 1.5$) and 3.5 cells ($r = 3.5$) respectively.



a) Overall Accuracy and Kappa Coefficient



b) Producer's Accuracy



c) User's Accuracy

Figure 5 - Simulation accuracies of the model in 1996 under various neighbourhood scales. For the small neighbourhood $r = 1.5$; for the moderate neighbourhood $r = 2.5$ and for the large neighbourhood $r = 3.5$.

To examine the model's behaviour over time, the results generated by the model from 1971 to 1996 under the three different neighbourhood scales were also summarised and compared against each other and with the actual urban development of Sydney over the same period of time (Figure 6). With the moderate neighbourhood scale, the model generated development that closely reflects the actual urban development of Sydney.

However, large differences between the simulated result under the small or the large neighbourhood scale and the actual urban development exist. These differences were especially significant for the urban and partly-urban categories around the year 1981 (Figure 6 a and b).

Put differently, as the urban development is controlled by the locally defined transition rules in a cellular automaton, the impact a cell can receive from its neighbourhood is limited to a certain distance. In the case of the urban development of Sydney, this distance is around 625 metres, corresponding to a local block or neighbourhood community.

This is why increasing the scale of the neighbourhood resulted in extended areas that drive the development of the cell. Yet this impact may not exist in actual urban development; and so the model generated over-development. On the other hand, decreasing the neighbourhood scale resulted in the reduction of areas that can drive the development of the cell, and so the model generated scenarios of under-development.

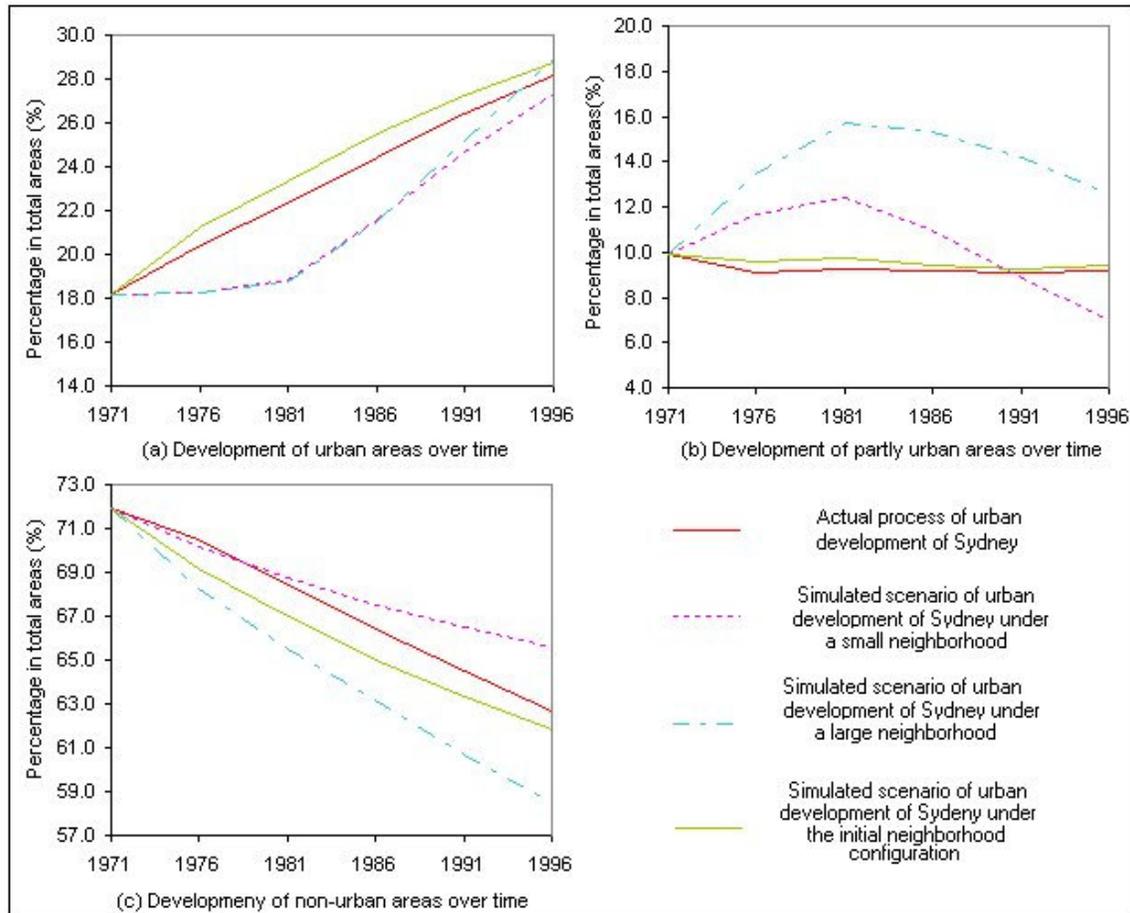


Figure 6 - Simulated urban development of Sydney, under various neighbourhood scales, over time. Whereas the model with the moderate neighbourhood generated similar development to actual urban development, significant differences exist between simulated and actual results when the small or large neighbourhood scales were used, especially around the year 1981.

4. Conclusion

The introduction of cellular automata modelling to urban studies is an applied science, and the scale of neighbourhood plays an important role in the configuration of the model and its performance. Although previous researchers applied both large and small neighbourhood scales in modelling urban growth using the cellular automata approach, no particular validation of the scale of neighbourhood has been explored.

Recent studies by Menard et al. (2005) demonstrate that the neighbourhood configuration of a geographical cellular automaton has less influence on the simulation outcome. However, our results show that different neighbourhood scales do have an impact on the performance and outcome of a CA model to a large extent, which is in accordance with findings by Kocabas et al. (2006).

Through the comparative studies of various neighbourhood scales on the model's behaviour and outcome, a moderate neighbourhood scale, the radius of which corresponds to a local block/neighbourhood community has been identified as the most suitable configuration to model the urban development of Sydney. When applying this model to simulate the process of urban development of another city, this neighbourhood scale may not be applicable and research will need to be conducted to search for a suitable neighbourhood scale.

However, as the neighbourhood scale is one of the fundamental elements in an urban cellular automaton, it is important to understand the scale of neighbourhood by which a cell can be affected and examine the variation of such neighbourhood scale on the model's performance and outcome. This paper provides a useful reference in searching for a suitable neighbourhood scale to model the process of urban development.

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